Coefficient estimates for families of bi-univalent functions defined by Ruscheweyh derivative operator

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Abstract. The main purpose of this manuscript is to find upper bounds for the second and third Taylor-Maclaurin coefficients for two families of holomorphic and bi-univalent functions associated with Ruscheweyh derivative operator. Further, we point out certain special cases for our results.

1. Introduction

Indicate by $\mathcal{A}$ the collection of all holomorphic functions in the open unit disk

$$U = \{z \in \mathbb{C} : |z| < 1\},$$

that have the form

$$(1) \qquad f(z) = z + \sum_{n=2}^{\infty} a_n z^n.$$

Further, assume that $\mathcal{S}$ stands for the sub-collection of the set $\mathcal{A}$ containing of functions in U satisfying (1) which are univalent in U.

For a function $f \in \mathcal{A}$ defined by (1), the Ruscheweyh derivative operator $\mathcal{R}^{\delta} : \mathcal{A} \longrightarrow \mathcal{A}$ (see [14]) is defined as follows:

$$\mathcal{R}^{\delta} f(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(\delta + n)}{(n-1)! \, \Gamma(\delta + 1)} a_n z^n,$$

$$(\delta \in \mathbb{N}_0 = \{0, 1, 2, \ldots\} := \mathbb{N} \cup \{0\}, \ z \in U).$$

According to the Koebe One-Quarter Theorem (see [6]) every function $f \in \mathcal{S}$ has an inverse f^{-1} defined by

$$f^{-1}(f(z)) = z \quad (z \in U) \quad \text{and} \quad f(f^{-1}(w)) = w \quad (|w| < r_0(f), \ r_0(f) \geq \tfrac{1}{4}).$$

For the inverse function f^{-1}, we have

(2)
$$\begin{aligned} g(w) = f^{-1}(w) &= w - a_2 w^2 + \left(2a_2^2 - a_3\right) w^3 \\ &\quad - \left(5a_2^3 - 5a_2 a_3 + a_4\right) w^4 + \cdots . \end{aligned}$$

For $f \in \mathcal{A}$, if both f and f^{-1} are univalent in U, we say that f is bi-univalent function in U. We indicate by Σ the family of bi-univalent functions in U given by (1). In fact, Srivastava et al. [22] have actually revived the study of holomorphic and bi-univalent functions in recent years, it was followed by such works as those by Frasin and Aouf [8], Murugusundaramoorthy et al. [13], Srivastava and Wanas [25] and others (see, for example [1, 3, 4, 5, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 21, 23, 14, 24, 26, 27, 28, 29, 30, 31, 32, 33]). We notice that the family Σ is not empty. Some examples of functions in the family Σ are

$$\frac{z}{1-z}, \quad \frac{1}{2} \log \left(\frac{1+z}{1-z}\right) \quad \text{and} \quad -\log(1-z),$$

with the corresponding inverse functions

$$\frac{w}{1+w}, \quad \frac{e^{2w}-1}{e^{2w}+1} \quad \text{and} \quad \frac{e^w-1}{e^w},$$

respectively. Other common examples of functions is not a member of Σ are

$$z - \frac{z^2}{2} \quad \text{and} \quad \frac{z}{1-z^2}.$$

Until now, the coefficient estimate problem for each of the following Taylor-Maclaurin coefficients $|a_n|$ $(n = 3, 4, \ldots)$ for functions $f \in \Sigma$ is still an open problem.

We require the following lemma that will be used to prove our main results.

Lemma 1 ([6]). *If $h \in \mathcal{P}$, then $|c_k| \le 2$ for each $k \in \mathbb{N}$, where $\mathcal{P}$ is the family of all functions h holomorphic in U for which*

$$\Re(h(z)) > 0 \quad (z \in U),$$

where

$$h(z) = 1 + c_1 z + c_2 z^2 + \cdots \quad (z \in U).$$

2. Coefficient Estimates for the Functions Family
$$\mathcal{Q}_\Sigma(\gamma, \mu, \lambda, \delta; \alpha)$$

Throughout this section, we suppose that

$$\mu \ge 1, \ \gamma \ge 0, \ 0 < \alpha \le 1, \ \delta \in \mathbb{N}_0 \text{ and } \lambda \in \mathbb{C} \backslash \{0\}.$$

Definition 1. A function $f \in \Sigma$ given by (1) is called in the family $\mathcal{Q}_\Sigma(\gamma, \mu, \lambda, \delta; \alpha)$ if it fulfills the conditions:

(3) $$\left| \arg \left(1 + \frac{1}{\lambda} \left[(1-\mu) \frac{\mathcal{R}^\delta f(z)}{z} + \mu \left(\mathcal{R}^\delta f(z)\right)' + \gamma z \left(\mathcal{R}^\delta f(z)\right)'' - 1 \right] \right) \right| < \frac{\alpha \pi}{2}$$

and

(4) $\left| \arg \left(1 + \frac{1}{\lambda} \left[(1-\mu) \frac{\mathcal{R}^\delta g(w)}{w} + \mu \left(\mathcal{R}^\delta g(w) \right)' + \gamma w \left(\mathcal{R}^\delta g(w) \right)'' - 1 \right] \right) \right| < \frac{\alpha\pi}{2},$

where $z, w \in U$ and $g = f^{-1}$ is given by (2).

Remark 1. It should be remarked that the family $\mathcal{Q}_\Sigma(\gamma, \mu, \lambda, \delta; \alpha)$ is a generalization of well-known families consider earlier. These families are:

(1) for $\delta = 0$ and $\mu = \lambda = 1$, the family $\mathcal{Q}_\Sigma(\gamma, \mu, \lambda, \delta; \alpha)$ reduce to the family $\mathcal{H}_\Sigma(\alpha, \gamma)$ which was considered by Frasin [7];

(2) for $\gamma = \delta = 0$ and $\lambda = 1$, the family $\mathcal{Q}_\Sigma(\gamma, \mu, \lambda, \delta; \alpha)$ reduce to the family $\mathcal{B}_\Sigma(\alpha, \mu)$ which was given by Frasin and Aouf [8];

(3) for $\gamma = \delta = 0$ and $\mu = \lambda = 1$, the family $\mathcal{Q}_\Sigma(\gamma, \mu, \lambda, \delta; \alpha)$ reduce to the family $\mathcal{H}_\Sigma^\alpha$ which was investigated by Srivastava et al. [22].

Theorem 1. Let $f \in \mathcal{Q}_\Sigma(\gamma, \mu, \lambda, \delta; \alpha)$ be given by (1). Then

$$|a_2| \leq \frac{2\alpha |\lambda|}{\sqrt{\left| \alpha\lambda(\delta+2)(\delta+1)(1+2\mu+6\gamma) + (1-\alpha)(\delta+1)^2 (1+\mu+2\gamma)^2 \right|}}$$

and

$$|a_3| \leq \frac{4\alpha^2 |\lambda|^2}{(\delta+1)^2 (1+\mu+2\gamma)^2} + \frac{4\alpha |\lambda|}{(\delta+2)(\delta+1)(1+2\mu+6\gamma)}.$$

Proof. In the light of the conditions (3) and (4), we deduce that

(5) $1 + \frac{1}{\lambda} \left[(1-\mu) \frac{\mathcal{R}^\delta f(z)}{z} + \mu \left(\mathcal{R}^\delta f(z) \right)' + \gamma z \left(\mathcal{R}^\delta f(z) \right)'' - 1 \right] = [p(z)]^\alpha$

and

(6) $1 + \frac{1}{\lambda} \left[(1-\mu) \frac{\mathcal{R}^\delta g(w)}{w} + \mu \left(\mathcal{R}^\delta g(w) \right)' + \gamma w \left(\mathcal{R}^\delta g(w) \right)'' - 1 \right] = [q(w)]^\alpha,$

where $g = f^{-1}$ is given by (2) and p, q in $\mathcal{P}$ have the following series representations:

(7) $$p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \cdots$$

and

(8) $$q(w) = 1 + q_1 w + q_2 w^2 + q_3 w^3 + \cdots.$$

Comparing the corresponding coefficients of (5) and (6) yields

(9) $$\frac{(\delta+1)(1+\mu+2\gamma)}{\lambda} a_2 = \alpha p_1,$$

and

(10) $$\frac{(\delta+2)(\delta+1)(1+2\mu+6\gamma)}{2\lambda} a_3 = \alpha p_2 + \frac{\alpha(\alpha-1)}{2} p_1^2,$$

(11)
$$-\frac{(\delta+1)(1+\mu+2\gamma)}{\lambda}a_2 = \alpha q_1$$

and

(12)
$$\frac{(\delta+2)(\delta+1)(1+2\mu+6\gamma)}{2\lambda}\left(2a_2^2 - a_3\right) = \alpha q_2 + \frac{\alpha(\alpha-1)}{2}q_1^2.$$

In view of (9) and (11), we conclude that

(13)
$$p_1 = -q_1$$

and

(14)
$$\frac{2(\delta+1)^2(1+\mu+2\gamma)^2}{\lambda^2}a_2^2 = \alpha^2(p_1^2 + q_1^2).$$

Also, by using (10) and (12), together with (14), we find that

$$\frac{(\delta+2)(\delta+1)(1+2\mu+6\gamma)}{\lambda}a_2^2 = \alpha(p_2 + q_2) + \frac{\alpha(\alpha-1)}{2}\left(p_1^2 + q_1^2\right)$$

$$= \alpha(p_2 + q_2) + \frac{(\alpha-1)(\delta+1)^2(1+\mu+2\gamma)^2}{\alpha\lambda^2}a_2^2.$$

Further computations show that

(15)
$$a_2^2 = \frac{\alpha^2\lambda^2(p_2 + q_2)}{\alpha\lambda(\delta+2)(\delta+1)(1+2\mu+6\gamma) + (1-\alpha)(\delta+1)^2(1+\mu+2\gamma)^2},$$

By taking the absolute value of (15) and applying Lemma 1 for the coefficients p_2 and q_2, we have

$$|a_2| \leq \frac{2\alpha|\lambda|}{\sqrt{\left|\alpha\lambda(\delta+2)(\delta+1)(1+2\mu+6\gamma) + (1-\alpha)(\delta+1)^2(1+\mu+2\gamma)^2\right|}}.$$

To determinate the bound on $|a_3|$, by subtracting (12) from (10), we get

(16)
$$\frac{(\delta+2)(\delta+1)(1+2\mu+6\gamma)}{\lambda}\left(a_3 - a_2^2\right) = \alpha(p_2 - q_2) + \frac{\alpha(\alpha-1)}{2}\left(p_1^2 - q_1^2\right).$$

Now, substituting the value of a_2^2 from (14) into (16) and using (13), we deduce that

(17)
$$a_3 = \frac{\alpha^2\lambda^2\left(p_1^2 + q_1^2\right)}{2(\delta+1)^2(1+\mu+2\gamma)^2} + \frac{\alpha\lambda(p_2 - q_2)}{(\delta+2)(\delta+1)(1+2\mu+6\gamma)}.$$

Taking the absolute value of (17) and applying Lemma 1 once again for the coefficients p_1, p_2, q_1 and q_2, it follows that

$$|a_3| \leq \frac{4\alpha^2|\lambda|^2}{(\delta+1)^2(1+\mu+2\gamma)^2} + \frac{4\alpha|\lambda|}{(\delta+2)(\delta+1)(1+2\mu+6\gamma)}. \qquad \Box$$

Remark 2. In Theorem 1, if we choose

(1) $\delta = 0$ and $\mu = \lambda = 1$, then we have the results obtained by Frasin [7, Theorem 2.2];

(2) $\gamma = \delta = 0$ and $\lambda = 1$, then we have the results obtained by Frasin and Aouf [8, Theorem 2.2];

(3) $\gamma = \delta = 0$ and $\mu = \lambda = 1$, then we obtain the results obtained by Srivastava et al. [22, Theorem 1].

3. Coefficient Estimates for the Functions Family $\mathcal{Q}_\Sigma^*(\gamma, \mu, \lambda, \delta; \beta)$

Throughout this section, we suppose that

$$\mu \geq 1, \ \gamma \geq 0, \ 0 \leq \beta < 1, \ \delta \in \mathbb{N}_0 \text{ and } \lambda \in \mathbb{C} \backslash \{0\}.$$

Definition 2. A function $f \in \Sigma$ given by (1) is called in the family $\mathcal{Q}_\Sigma^*(\gamma, \mu, \lambda, \delta; \beta)$ if it fulfills the conditions

$$(18) \quad \Re \left\{ 1 + \frac{1}{\lambda} \left[(1-\mu) \frac{\mathcal{R}^\delta f(z)}{z} + \mu \left(\mathcal{R}^\delta f(z) \right)' + \gamma z \left(\mathcal{R}^\delta f(z) \right)'' - 1 \right] \right\} > \beta$$

and

$$(19) \quad \Re \left\{ 1 + \frac{1}{\lambda} \left[(1-\mu) \frac{\mathcal{R}^\delta g(w)}{w} + \mu \left(\mathcal{R}^\delta g(w) \right)' + \gamma w \left(\mathcal{R}^\delta g(w) \right)'' - 1 \right] \right\} > \beta,$$

where $z, w \in U$ and $g = f^{-1}$ is given by (2).

Remark 3. It should be remarked that the family $\mathcal{Q}_\Sigma^*(\gamma, \mu, \lambda, \delta; \beta)$ is a generalization of well-known families consider earlier. These families are:

(1) for $\delta = 0$ and $\lambda = 1$, the family $\mathcal{Q}_\Sigma^*(\gamma, \mu, \lambda, \delta; \beta)$ reduce to the family $\mathcal{N}_\Sigma(\beta, \mu, \gamma)$ which was defined by Bulut [2];
(2) for $\delta = 0$ and $\mu = \lambda = 1$, the family $\mathcal{Q}_\Sigma^*(\gamma, \mu, \lambda, \delta; \beta)$ reduce to the family $\mathcal{H}_\Sigma(\gamma, \beta)$ which was considered by Frasin [7];
(3) for $\gamma = \delta = 0$ and $\lambda = 1$, the family $\mathcal{Q}_\Sigma^*(\gamma, \mu, \lambda, \delta; \beta)$ reduce to the family $\mathcal{B}_\Sigma(\beta, \mu)$ which was given by Frasin and Aouf [8];
(4) for $\gamma = \delta = 0$ and $\mu = \lambda = 1$, the family $\mathcal{Q}_\Sigma^*(\gamma, \mu, \lambda, \delta; \beta)$ reduce to the family $\mathcal{H}_\Sigma(\beta)$ which was investigated by Srivastava et al. [22].

Theorem 2. Let $f \in \mathcal{Q}_\Sigma^*(\gamma, \mu, \lambda, \delta; \beta)$ be given by (1). Then

$$|a_2| \leq \begin{cases} 2\sqrt{\frac{|\lambda|(1-\beta)}{(\delta+2)(\delta+1)(1+2\mu+6\gamma)}}, & 0 \leq \beta \leq 1 - \frac{(\delta+1)(1+\mu+2\gamma)^2}{|\lambda|(\delta+2)(1+2\mu+6\gamma)}, \\[4mm] \frac{2|\lambda|(1-\beta)}{(\delta+1)(1+\mu+2\gamma)}, & 1 - \frac{(\delta+1)(1+\mu+2\gamma)^2}{|\lambda|(\delta+2)(1+2\mu+6\gamma)} \leq \beta < 1 \end{cases}$$

and

$$|a_3| \leq \frac{4\,|\lambda|\,(1-\beta)}{(\delta+2)(\delta+1)\,(1+2\mu+6\gamma)}.$$

Proof. In the light of the conditions (18) and (19), there are $p, q \in \mathcal{P}$ such that

$$(20) \quad 1 + \frac{1}{\lambda} \left[(1 - \mu) \frac{\mathcal{R}^\delta f(z)}{z} + \mu \left(\mathcal{R}^\delta f(z) \right)' + \gamma z \left(\mathcal{R}^\delta f(z) \right)'' - 1 \right]$$
$$= \beta + (1 - \beta) p(z)$$

and

$$(21) \quad 1 + \frac{1}{\lambda} \left[(1 - \mu) \frac{\mathcal{R}^\delta g(w)}{w} + \mu \left(\mathcal{R}^\delta g(w) \right)' + \gamma w \left(\mathcal{R}^\delta g(w) \right)'' - 1 \right]$$
$$= \beta + (1 - \beta) q(w),$$

where $g = f^{-1}$ is given by (2), $p(z)$ and $q(w)$ have the forms (7) and (8), respectively. Comparing the corresponding coefficients in (20) and (21) yields

$$(22) \quad \frac{(\delta + 1)(1 + \mu + 2\gamma)}{\lambda} a_2 = (1 - \beta) p_1,$$

$$(23) \quad \frac{(\delta + 2)(\delta + 1)(1 + 2\mu + 6\gamma)}{2\lambda} a_3 = (1 - \beta) p_2,$$

$$(24) \quad -\frac{(\delta + 1)(1 + \mu + 2\gamma)}{\lambda} a_2 = (1 - \beta) q_1$$

and

$$(25) \quad \frac{(\delta + 2)(\delta + 1)(1 + 2\mu + 6\gamma)}{2\lambda} \left(2a_2^2 - a_3 \right) = (1 - \beta) q_2.$$

From (22) and (24), we get

$$p_1 = -q_1$$

and

$$(26) \quad \frac{2(\delta + 1)^2 (1 + \mu + 2\gamma)^2}{\lambda^2} a_2^2 = (1 - \beta)^2 \left(p_1^2 + q_1^2 \right).$$

Adding (23) and (25), we obtain

$$(27) \quad \frac{(\delta + 2)(\delta + 1)(1 + 2\mu + 6\gamma)}{\lambda} a_2^2 = (1 - \beta)(p_2 + q_2).$$

Hence, we find from (26) and (27) that

$$a_2^2 = \frac{\lambda^2 (1 - \beta)^2 \left(p_1^2 + q_1^2 \right)}{2(\delta + 1)^2 (1 + \mu + 2\gamma)^2}$$

and

$$a_2^2 = \frac{\lambda(1 - \beta)(p_2 + q_2)}{(\delta + 2)(\delta + 1)(1 + 2\mu + 6\gamma)},$$

respectively. By applying Lemma 1 for the coefficients p_2 and q_2, we deduce that

$$|a_2| \leq \frac{2|\lambda|(1 - \beta)}{(\delta + 1)(1 + \mu + 2\gamma)}$$

and

$$|a_2| \leq 2\sqrt{\frac{|\lambda|\,(1-\beta)}{(\delta+2)(\delta+1)\,(1+2\mu+6\gamma)}},$$

respectively. To determinate the bound on $|a_3|$, by subtracting (25) from (23), we get

$$\frac{(\delta+2)(\delta+1)\,(1+2\mu+6\gamma)}{\lambda}\left(a_3 - a_2^2\right) = (1-\beta)\,(p_2 - q_2),$$

or equivalently

(28)
$$a_3 = a_2^2 + \frac{\lambda(1-\beta)\,(p_2 - q_2)}{(\delta+2)(\delta+1)\,(1+2\mu+6\gamma)}.$$

Substituting the value of a_2^2 from (26) and (27) into (28), it follows that

$$a_3 = \frac{\lambda^2\,(1-\beta)^2\,(p_1^2 + q_1^2)}{2\,(\delta+1)^2\,(1+\mu+2\gamma)^2} + \frac{\lambda(1-\beta)\,(p_2 - q_2)}{(\delta+2)(\delta+1)\,(1+2\mu+6\gamma)}$$

and

$$a_3 = \frac{2\lambda(1-\beta)p_2}{(\delta+2)(\delta+1)\,(1+2\mu+6\gamma)},$$

respectively. By applying Lemma 1 once again for the coefficients p_1, p_2, q_1 and q_2, we deduce that

$$|a_3| \leq \frac{4\,|\lambda|^2\,(1-\beta)^2}{(\delta+1)^2\,(1+\mu+2\gamma)^2} + \frac{4\,|\lambda|\,(1-\beta)}{(\delta+2)(\delta+1)\,(1+2\mu+6\gamma)}$$

and

$$|a_3| \leq \frac{4\,|\lambda|\,(1-\beta)}{(\delta+2)(\delta+1)\,(1+2\mu+6\gamma)},$$

respectively. This completes the proof. $\qquad\square$

Remark 4. In Theorem 2, if we choose
(1) $\delta = 0$ and $\lambda = 1$, then we have the results obtained by Bulut [2, Theorem 3.4];
(2) $\delta = 0$ and $\mu = \lambda = 1$, then we have improvements of the results obtained by Frasin [7, Theorem 3.2];
(3) $\gamma = \delta = 0$ and $\lambda = 1$, then we have improvements of the results obtained by Frasin and Aouf [8, Theorem 3.2];
(4) $\gamma = \delta = 0$ and $\mu = \lambda = 1$, then we have improvements of the results obtained by Srivastava et al. [22, Theorem 2].

4. Conclusion

In this investigation, we have introduced and defined two a certain families of holomorphic and bi-univalent functions in the open unit disk U associated with Ruscheweyh derivative operator. We have then derived the initial coefficient estimations for functions belonging to these families. Further by specializing the parameters, several consequences of these families are mentioned.

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